Preliminary 3d-MHD modeling efforts for new liquid lithium dimes probe with HIMAG Code

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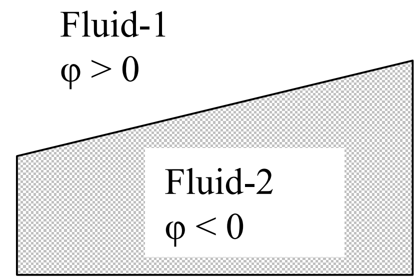
Problem Formulation - 1 : Fluid flow



Governing equations

Continuity
$$\frac{\partial u_i}{\partial x_i} = 0,$$

Levelset
$$\frac{\partial \varphi}{\partial t} + u_i \frac{\partial (\varphi)}{\partial x_i} = 0, \rightarrow \frac{\partial \varphi}{\partial \tau} = sign(\varphi_0)\left[1 - \left|\nabla(\varphi)\right|\right]$$

$$\rho = \rho_1 + (\rho_2 - \rho_1)H(\varphi) \quad , \quad \mu = \mu_1 + (\mu_2 - \mu_1)H(\varphi)$$

Momentum
$$\frac{\partial (u_i)}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \underbrace{\frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right)}_{\text{viscous}} + \underbrace{\frac{\vec{J} \times \vec{B}}{\rho}}_{\text{Lorentz}} + \underbrace{\vec{g}}_{\text{gravity}} + \underbrace{\frac{\sigma \kappa(\vec{x}) \vec{\nabla} \varphi}{\rho}}_{\text{Surface tension}}$$

A general projection methods for the incompressible Navier-Stokes equation

Projection I

$$\boldsymbol{A_v}\hat{\boldsymbol{v}} = \boldsymbol{r_v} - \boldsymbol{G}\,p^n$$

$$\tilde{\boldsymbol{v}} = \hat{\boldsymbol{v}} + \alpha\Delta t\,\boldsymbol{G}\,p^n$$

$$\alpha\Delta t\,\boldsymbol{DG}\,p^{n+1} = \boldsymbol{D}\tilde{\boldsymbol{v}}$$

$$\boldsymbol{v}^{\boldsymbol{n+1}} = \tilde{\boldsymbol{v}} - \alpha\Delta t\,\boldsymbol{G}\,p^{n+1}$$

Projection II

$$\boldsymbol{A_v}\hat{\boldsymbol{v}} = \boldsymbol{r_v} - \boldsymbol{G}\,p^n$$

$$\alpha\Delta t\,\boldsymbol{DG}\left(p^{n+1} - p^n\right) = \boldsymbol{D}\hat{\boldsymbol{v}}$$

$$\boldsymbol{v}^{\boldsymbol{n+1}} = \hat{\boldsymbol{v}} - \alpha\Delta t\,\boldsymbol{G}\left(p^{n+1} - p^n\right)$$

$$\boldsymbol{A_v} = \frac{1}{\Delta t}\left[\boldsymbol{I} - \frac{\Delta t}{2\mathrm{Re}}\boldsymbol{L}\right]$$

$$\boldsymbol{r_v} = \frac{1}{\Delta t}\left[\boldsymbol{I} + \frac{\Delta t}{2\mathrm{Re}}\boldsymbol{L}\right]\boldsymbol{v}^{\boldsymbol{n}} - \boldsymbol{N}^{\boldsymbol{n+\frac{1}{2}}}(\boldsymbol{v})$$

Second order accuracy with error as

$$\left((\Delta t)^2/(2\mathrm{Re})\right)\boldsymbol{LG}\left(\left(p^{n+1} - p^n\right)/\Delta t\right)$$

Bell, Colella and Glaz, 1989

Choi, Moin, 1994

Dukowicz, Dvinsky, 1992

Patankar, SIMPLE Method, 1980

$$\left(1/\Delta t + A_P^n\right)\hat{\boldsymbol{v}}_P = \boldsymbol{v}_P^n/\Delta t + \sum A_M^n\,\hat{\boldsymbol{v}}_M - \boldsymbol{G}\,p^{\boldsymbol{n}}$$

$$\boldsymbol{D}\left(\frac{\Delta t}{1 + A_P^n\Delta t}\boldsymbol{G}\left(p^{n+1} - p^n\right)\right) = \boldsymbol{D}\hat{\boldsymbol{v}}_P$$

$$\boldsymbol{v}_P^{n+1} = \hat{\boldsymbol{v}}_P - \frac{\Delta t}{1 + A_P^n\Delta t}\boldsymbol{G}\left(p^{n+1} - p^n\right)$$

Fractional time stepping scheme

Time advancement from n to n+1 (detailed solution description in report-1)

u,v,w,p, φ are stored at cell centers and a face normal velocity U
is stored at cell faces.

(i) Solve Crank-Nicholson discretized momentum equations for an intermediate state u*,v*,w*
(ii) Compute interpolated value U*
(iii) Solve pressure Poisson equation using U* to obtain p(n+1)
(iv) Obtain u,v,w, φ at n+1
(v) Obtain conservative velocity U^{n+1} after adjusting for pressure

Colocation is possible by means of momentum interpolation

Numerical implementation of the momentum solver

$$\frac{(\hat\rho\,\hat u_i)-(\rho\,u_i)^n}{\Delta t}+\frac{1}{2}\frac{\partial}{\partial x_j}\left(\hat\rho\,\hat u_i u_j^n+\hat\rho\,u_i^n\,\hat u_j\right)=-\frac{\partial p^n}{\partial x_i}+\frac{1}{2}\frac{\partial}{\partial x_j}\left(\hat\mu\frac{\partial}{\partial x_j}\left(\hat u_i+u_i^n\right)\right)$$

$$\frac{(\rho^*\,u_i^*)-(\rho\,\hat u_i)}{\Delta t}=\frac{\partial p^n}{\partial x_i}$$

$$\frac{\partial}{\partial x_i}\left(\frac{\partial p^{n+1}}{\partial x_i}\right)=\frac{1}{\Delta t}\frac{\partial \rho u_i^*}{\partial x_i}$$

$$\frac{(\rho^{n+1}\,u_i^{n+1})-(\rho\,u_i^*)}{\Delta t}=-\frac{\partial p^{n+1}}{\partial x_i}$$

$$u^*=\left(\overbrace{\delta\,\hat u+u^n}^{\hat u}+\frac{\Delta t}{\rho}\frac{\partial p^n}{\partial x}\right),$$

$$v^*=\left(\delta\,\hat v+v^n+\frac{\Delta t}{\rho}\frac{\partial p^n}{\partial y}\right),$$

$$w^*=\left(\delta\,\hat w+w^n+\frac{\Delta t}{\rho}\frac{\partial p^n}{\partial z}\right),$$

$$\frac{\delta\,\hat u_{ic}}{\Delta t}=-\frac{1}{2\Omega}\sum_{faces}\left(U^n\delta\,\hat u+u^n\delta\,\hat V^n+2u^nU^n\right)_f\Delta s-\frac{1}{\rho\,\Omega}\left(\sum_{faces}p_f\Delta s_x\right)+\frac{1}{2\rho\,\Omega}\sum_{faces}\frac{\partial}{\partial n}\hat\mu\left[\delta\,\hat u+2u^n\right]_f\Delta s$$

$$\frac{\delta\,\hat v_{ic}}{\Delta t}=-\frac{1}{2\Omega}\sum_{faces}\left(U^n\delta\,\hat v+v^n\delta\,\hat V^n+2v^nU^n\right)_f\Delta s-\frac{1}{\rho\,\Omega}\left(\sum_{faces}p_f\Delta s_y\right)+\frac{1}{2\rho\,\Omega}\sum_{faces}\frac{\partial}{\partial n}\hat\mu\left[\delta\,\hat v+2v^n\right]_f\Delta s$$

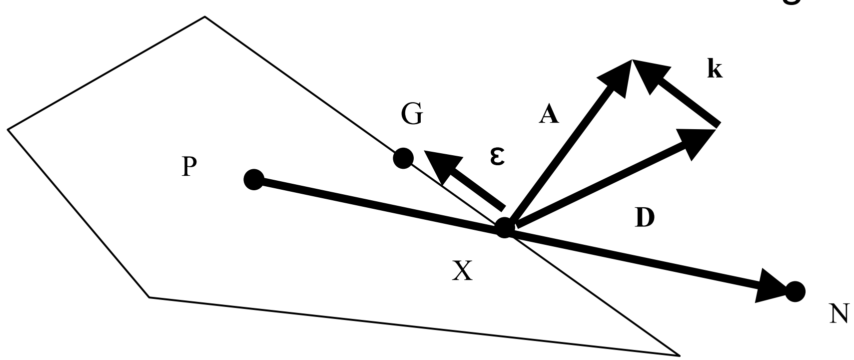
$$\frac{\delta\,\hat w_{ic}}{\Delta t}=-\frac{1}{2\Omega}\sum_{faces}\left(U^n\delta\,\hat w+w^n\delta\,\hat V^n+2w^nU^n\right)_f\Delta s-\frac{1}{\rho\,\Omega}\left(\sum_{faces}p_f\Delta s_z\right)+\frac{1}{2\rho\,\Omega}\sum_{faces}\frac{\partial}{\partial n}\hat\mu\left[\delta\,\hat w+2w^n\right]_f\Delta s$$

where $\delta\,\hat V^n=n_z\delta\,\hat w+n_y\delta\,\hat v+n_x\delta\,\hat u$

Pressure Poisson equation

$$\nabla \cdot \frac{\nabla p}{\rho} = \frac{\nabla \cdot \vec{V}^*}{\Delta t} \quad \Rightarrow \quad \frac{1}{\Omega} \sum_{faces} \frac{1}{\rho} \frac{\partial p}{\partial n} \Delta s = \frac{1}{\Omega \Delta t} \left(\sum_{\neq BC} U^* \Delta s + \sum_{= BC} U^{n+1} \Delta s \right)$$

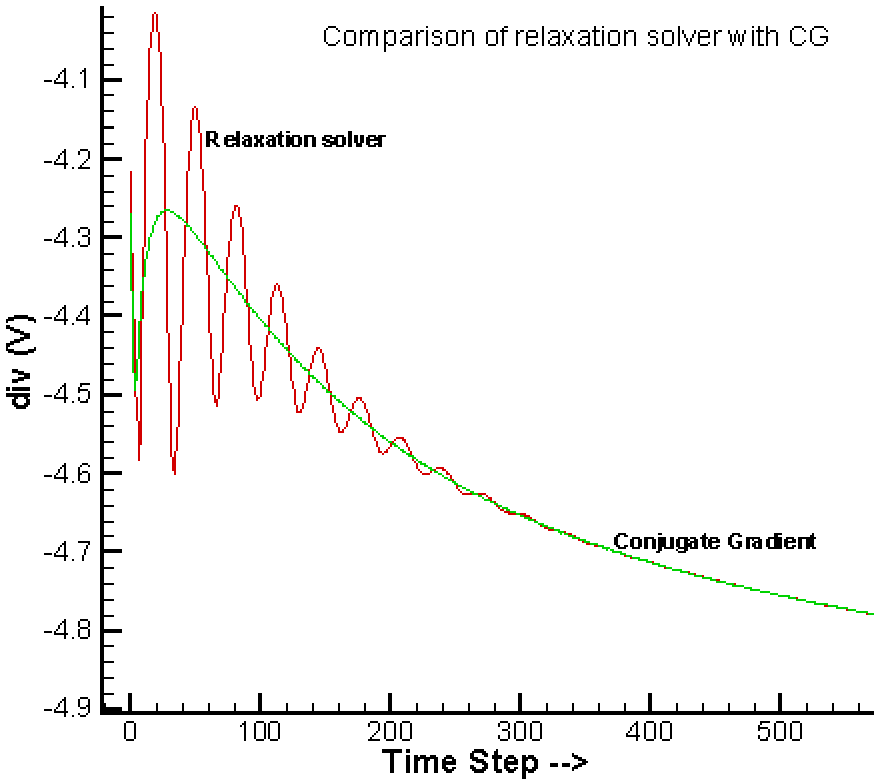
Nonorthogonal correction

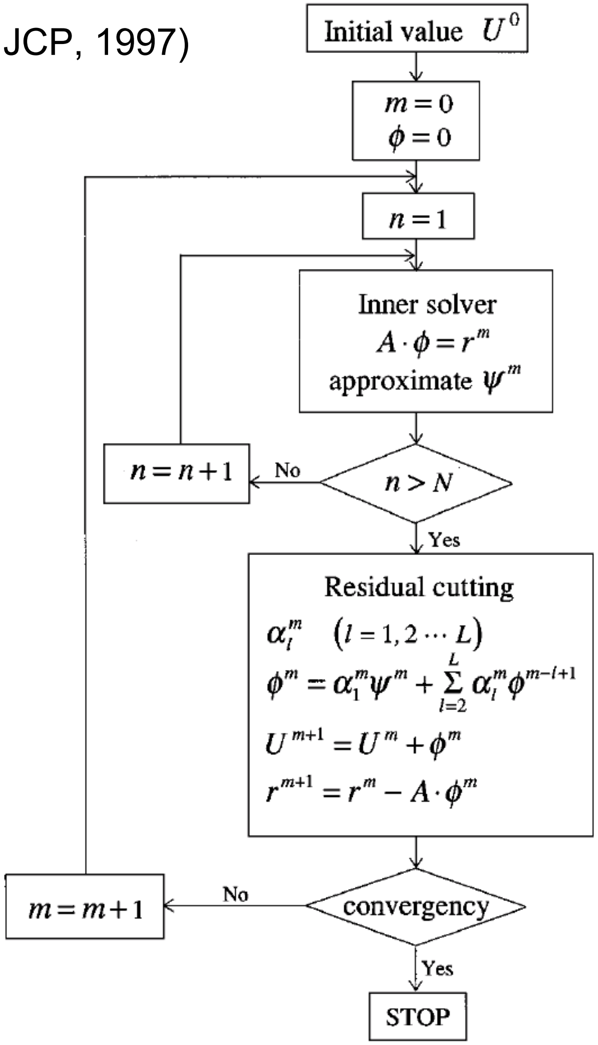


$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{n}} = \frac{1}{\Delta \boldsymbol{s}} \left(|\mathbf{D}| \frac{\boldsymbol{p}_N - \boldsymbol{p}_P}{|\mathbf{d}|} + \mathbf{k} \cdot (\nabla \boldsymbol{p})_{\boldsymbol{f}} \right)$$

Variations upon the CG technique

1. Variations in sweeping direction
2. Residual cutting technique (Tamura, Kikuchi, Takahashi, JCP, 1997)





Higher order accuracy

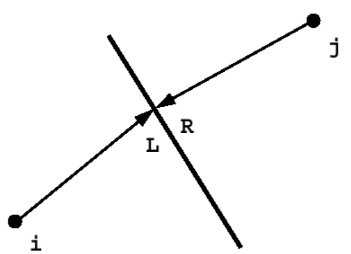
Gradient reconstruction using Barth's approach has been used successfully

$$U(\vec{r}) = u_0 + \nabla u_0 \cdot (\vec{r} - \vec{r}_0) \xrightarrow{\text{limiter}} U(\vec{r}) = u_0 + \Phi_0 \nabla u_0 \cdot (\vec{r} - \vec{r}_0)$$

$$u_t + \nabla \cdot f = 0$$

$$\downarrow$$

$$u^{n+1} = u^n - \frac{\Delta t}{\Omega} \sum_{faces} f_{face} \cdot \vec{n}$$



$$f_{face} = \frac{1}{2}(f_L + f_R) - \frac{1}{2}\left|\frac{df}{du}\right|(u_R - u_L)$$

$$u_j^{\min} = \min_{i \in \mathrm{N}_j}(u_0, u_i)$$

$$u_j^{\max} = \max_{i \in \mathrm{N}_j}(u_0, u_i)$$

$$\Phi_0 = \begin{cases} \min\left(1, \dfrac{u_j^{\max} - u_0}{U_{0i} - u_0}\right) & if \quad U_{0i} - u_0 > 0 \\ \min\left(1, \dfrac{u_j^{\min} - u_0}{U_{0i} - u_0}\right) & if \quad U_{0i} - u_0 < 0 \\ 1 & if \quad U_{0i} - u_0 = 0 \end{cases}$$

Heat Transfer

The heat equation is written as:

$$\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{t}} + \boldsymbol{u}_i \frac{\partial \boldsymbol{T}}{\partial \boldsymbol{x}_i} = \frac{\partial}{\partial \boldsymbol{x}_i}\left(\alpha \frac{\partial \boldsymbol{T}}{\partial \boldsymbol{x}_i}\right)$$

This is linearized in time, as:

$$\boldsymbol{T}^{n+1} = \boldsymbol{T}^{n} + \delta\, \boldsymbol{T}$$

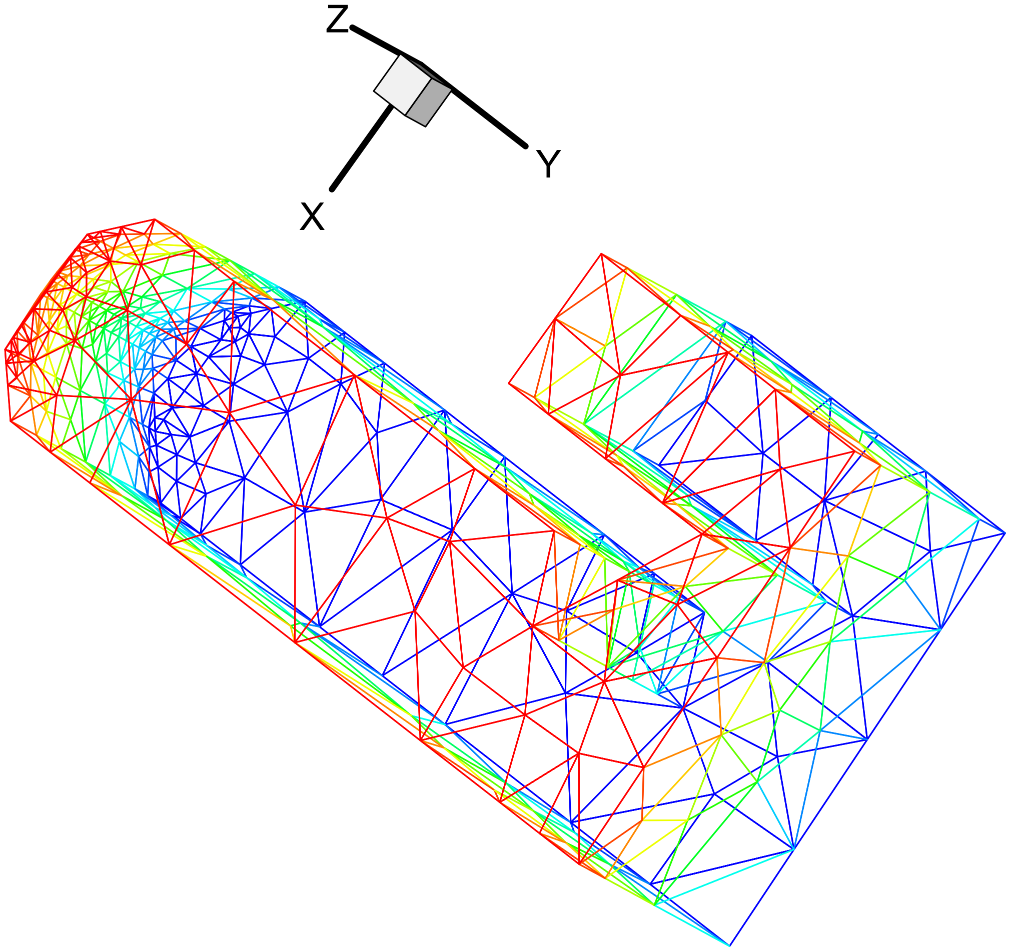
time averaging the temperature, the discretized energy equation becomes:

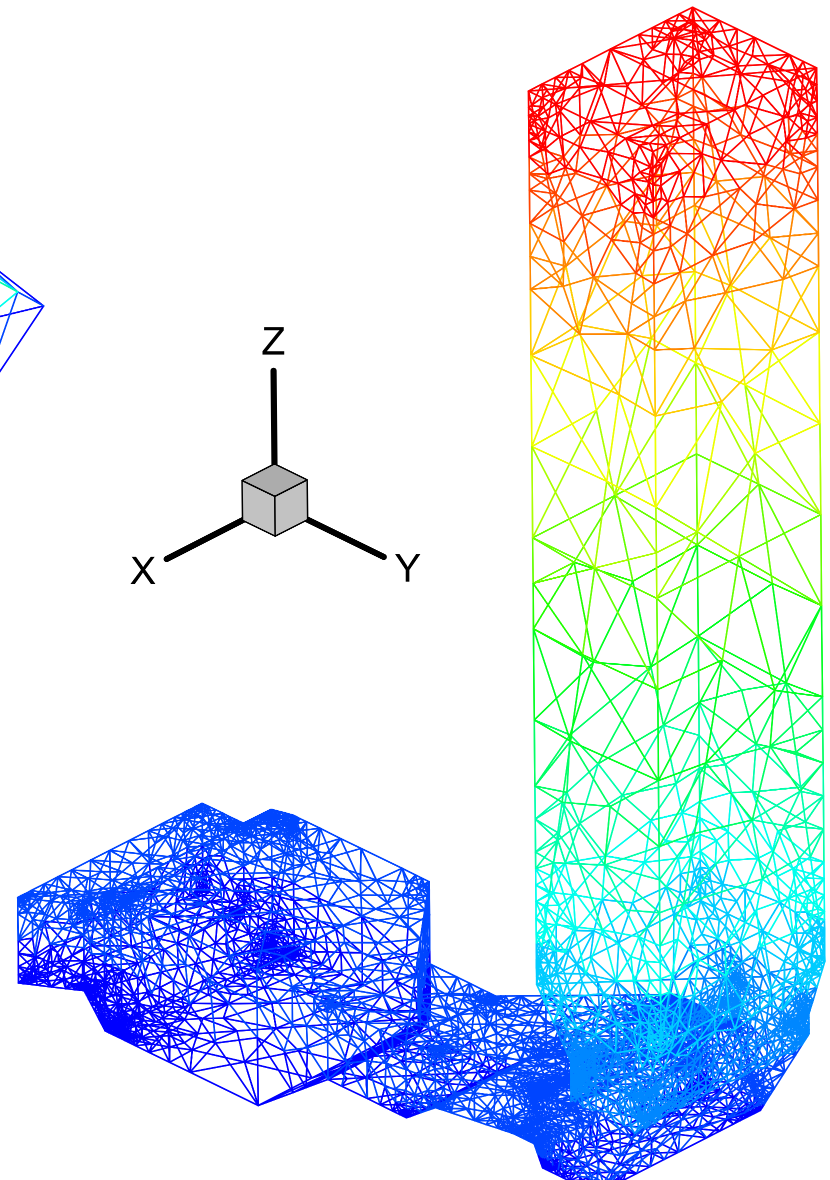
$$\delta\, \boldsymbol{T} = -\frac{\Delta \boldsymbol{t}}{\Omega} \sum_{faces} \boldsymbol{U}^{*}\left(\frac{2\boldsymbol{T}^{n} + \delta\, \boldsymbol{T}}{2}\right)\Delta \boldsymbol{s} + \frac{\Delta \boldsymbol{t}}{\Omega} \sum_{faces} \frac{\partial}{\partial \boldsymbol{n}} \alpha_{face}\left(\frac{2\boldsymbol{T}^{n} + \delta\, \boldsymbol{T}}{2}\right)\Delta \boldsymbol{s}$$

grouping terms, this becomes:

$$\delta T_{ic} = \frac{\dfrac{\Delta t}{\Omega}\left\{\displaystyle\sum_{faces}\left[-U T^{n}_{face} - \frac{1}{2}(1-f)U\delta\, T_{inb} + \alpha_{face}\left(\frac{T^{n}_{inb} - T^{n}_{ic}}{d} + \frac{\delta T_{inb}}{2d}\right)\right]\Delta s\right\}}{1 + \dfrac{\Delta t}{\Omega}\left\{\displaystyle\sum_{faces}\left[\frac{fU}{2} + \frac{\alpha_{face}}{2d}\right]\Delta s\right\}}$$

The effect of non-orthogonal terms are included in the same manner as in the pressure Poisson equation.


Z
X
Y

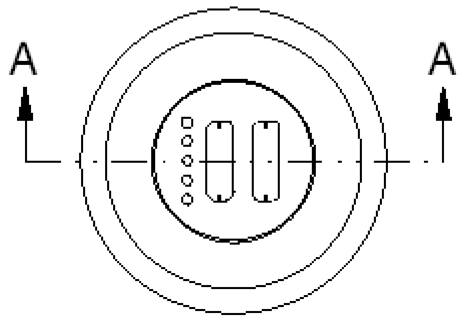

Z
X
Y

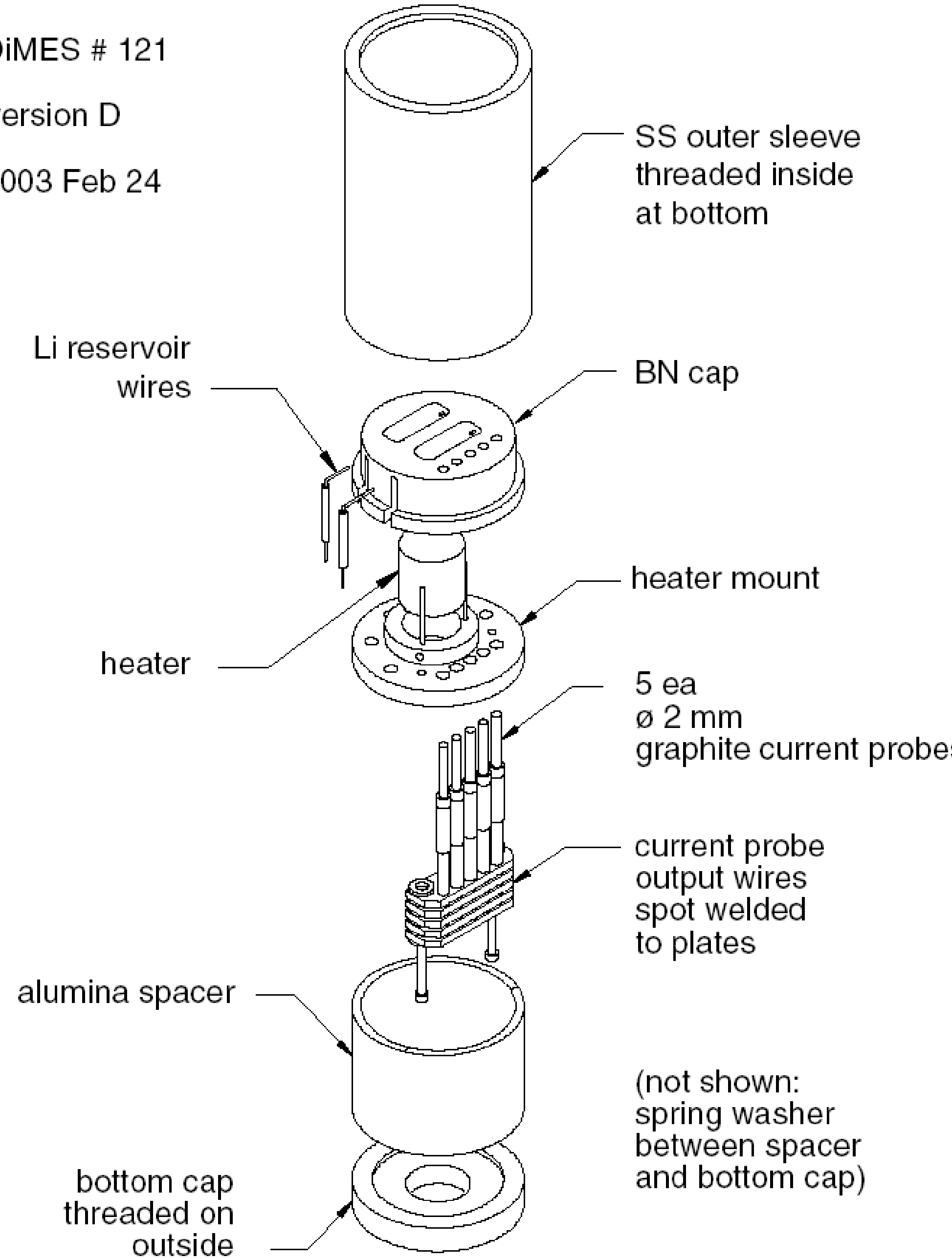
Geometry of DiMES Lithium Probe

DiMES # 121

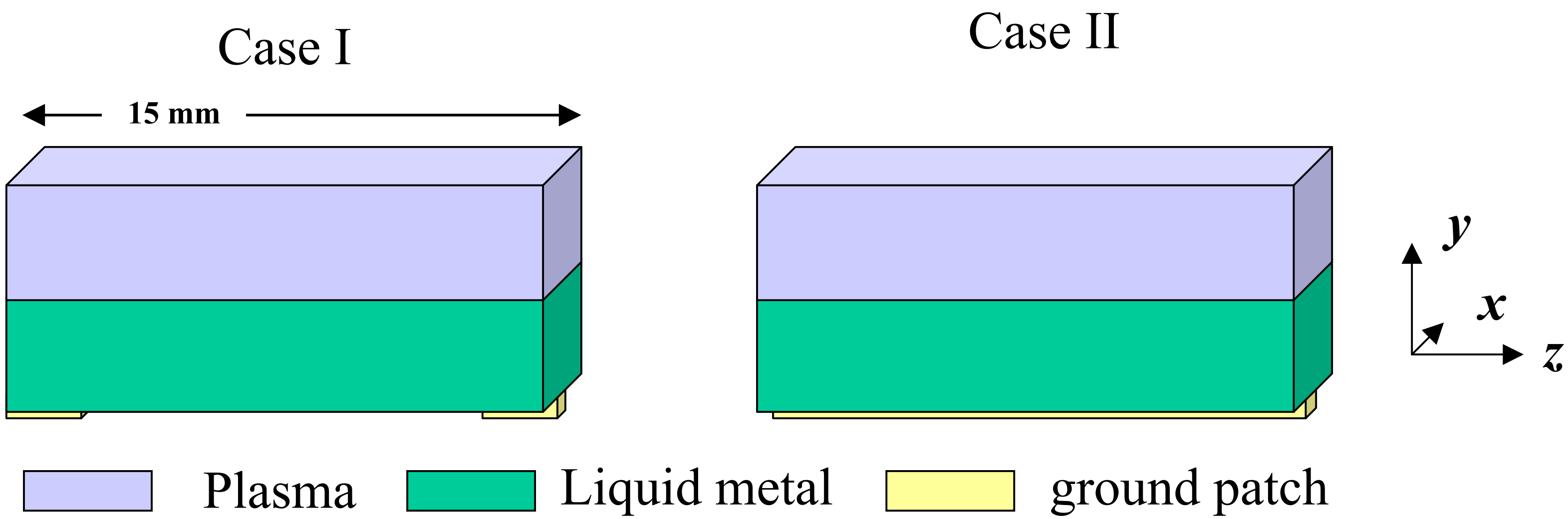
version D

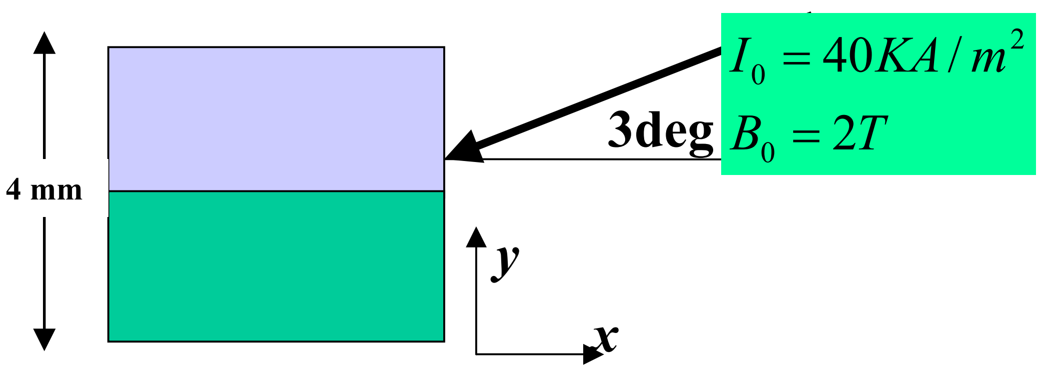
2003 Feb 24





Geometry of DIMES Lithium Slot





Lithium Properties:

$\rho = 485 kg/m^3$

$\upsilon = 8.5\times10^{-7} m^2/s$

$\sigma = 2.8\times10^{6}(mOhm)^{-1}$

$\sigma_s = 0.35 N/m$

DIMES Calculation using HIMAG

3-D with applied current and magnetic field:

J=40KA/m2, B=2, angle=3degree

Computational size: 4x5x15 mm (41x41x61)

Grounding patches included

two patches with sizes 0.5x0.5mm2 distributed on the bottom wall

Interfacial MHD flows

Non-Dimensional Parameters:

Characteristic length=5.0mm; velocity=sqrt(g*L)=0.22147m/s

Characteristic current density=1.24e6; magneto field=2T

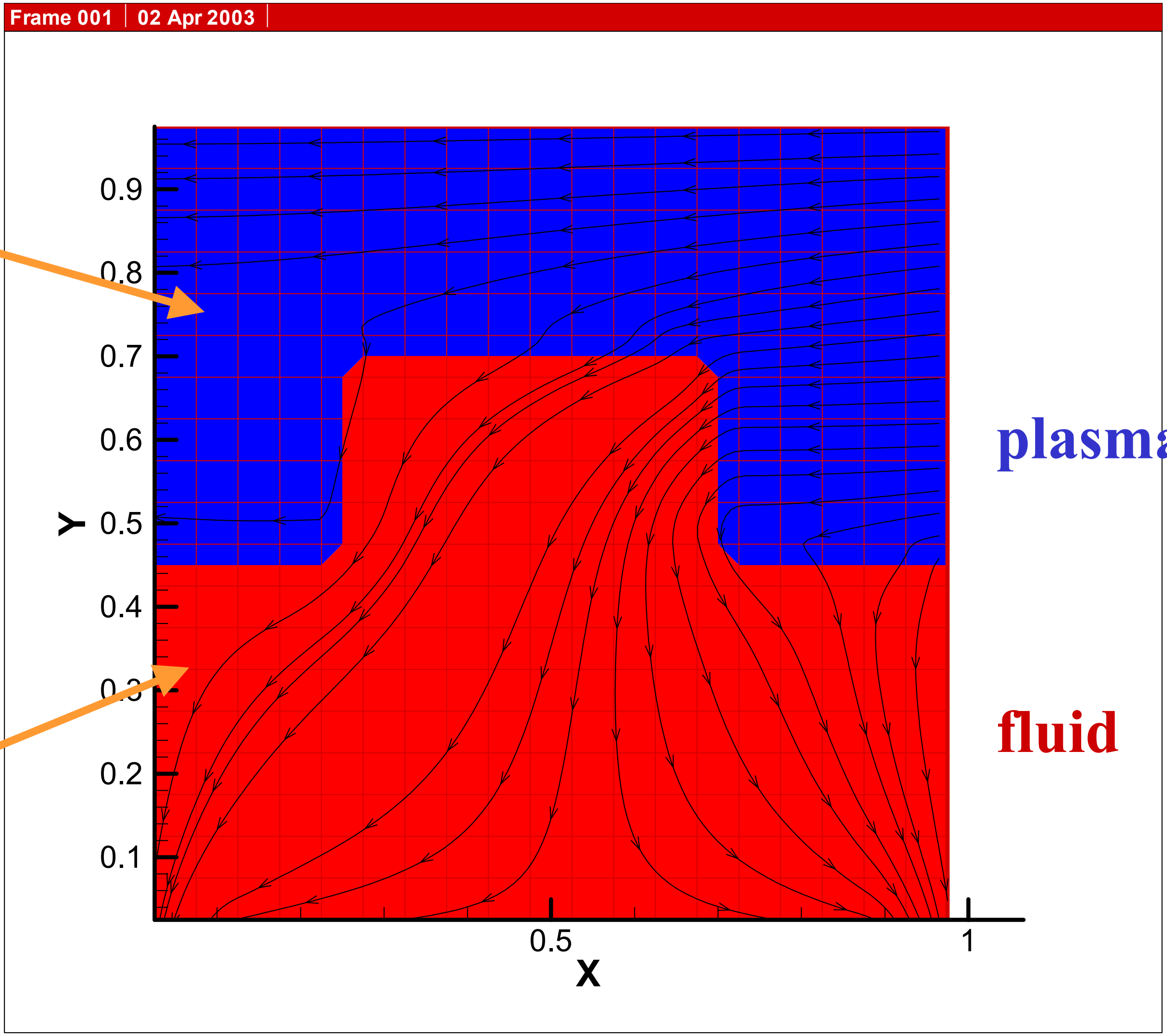
Fr=U*U/gL=1, We=0.339839, Re=1302.78,

Ha=824.136, N=521.346276

Modification of electric potential formulation to force current flow || to B

$$\left.\begin{aligned} j_x &= -\sigma_p \cos\theta \frac{\partial \phi}{\partial x} \\ j_y &= -\sigma_p \sin\theta \frac{\partial \phi}{\partial y} \\ j_z &= 0 \end{aligned}\right\}$$

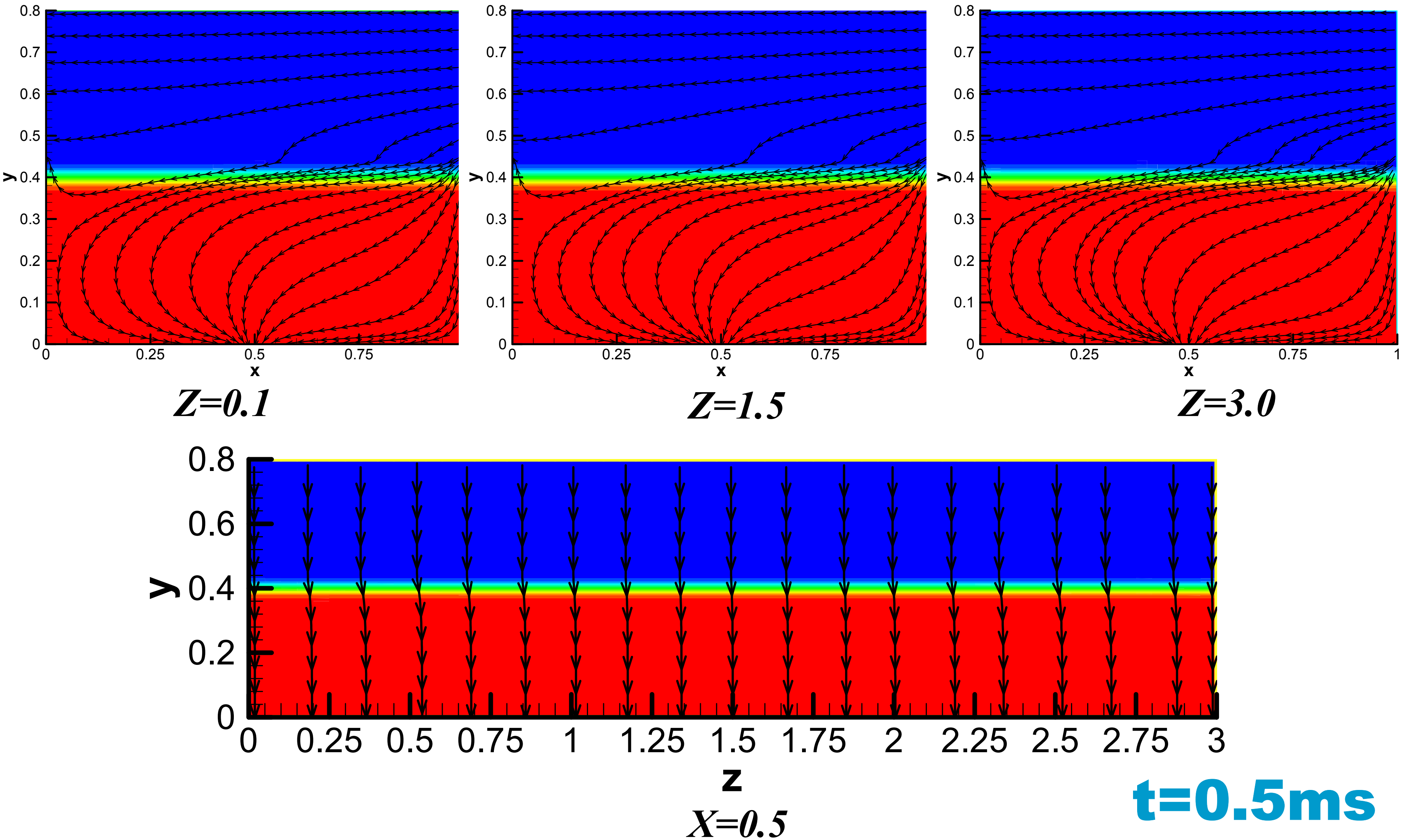
$$\left.\begin{aligned} j_x &= -\sigma_{LM} \frac{\partial \phi}{\partial x} \\ j_y &= -\sigma_{LM} \frac{\partial \phi}{\partial y} \\ j_z &= -\sigma_{LM} \frac{\partial \phi}{\partial z} \end{aligned}\right\}$$



plasma

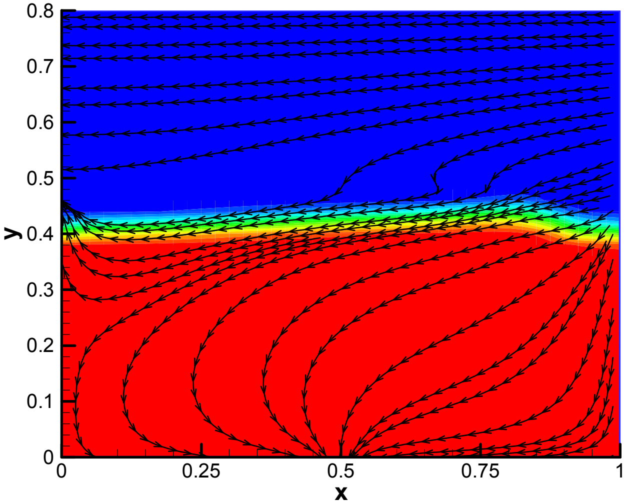
fluid

Center Grounding Strip: Density Contour and Current Streamline

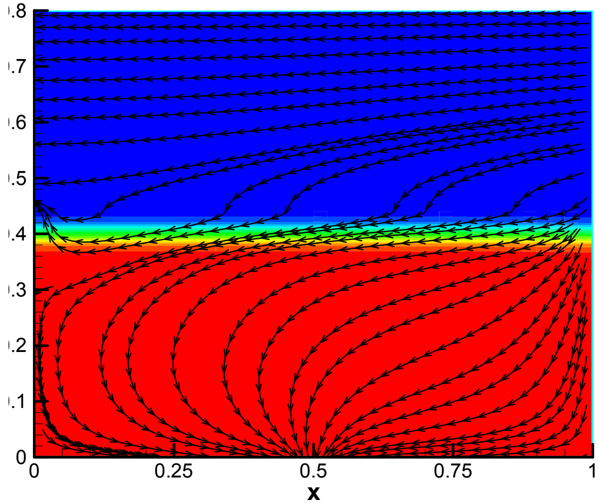


t=0.5ms

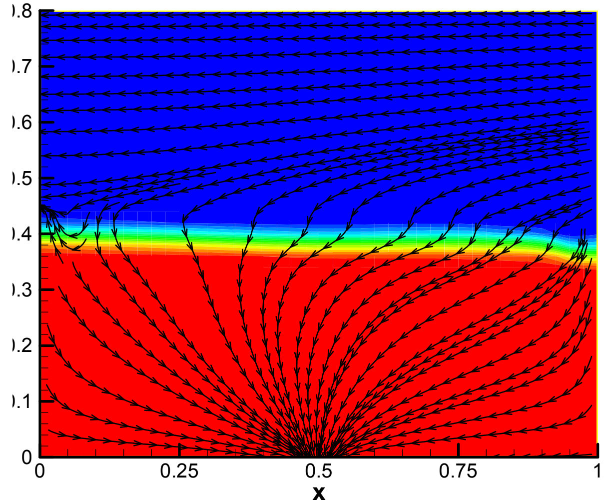
Center Grounding Strip: Density Contour and Electrical Streamline



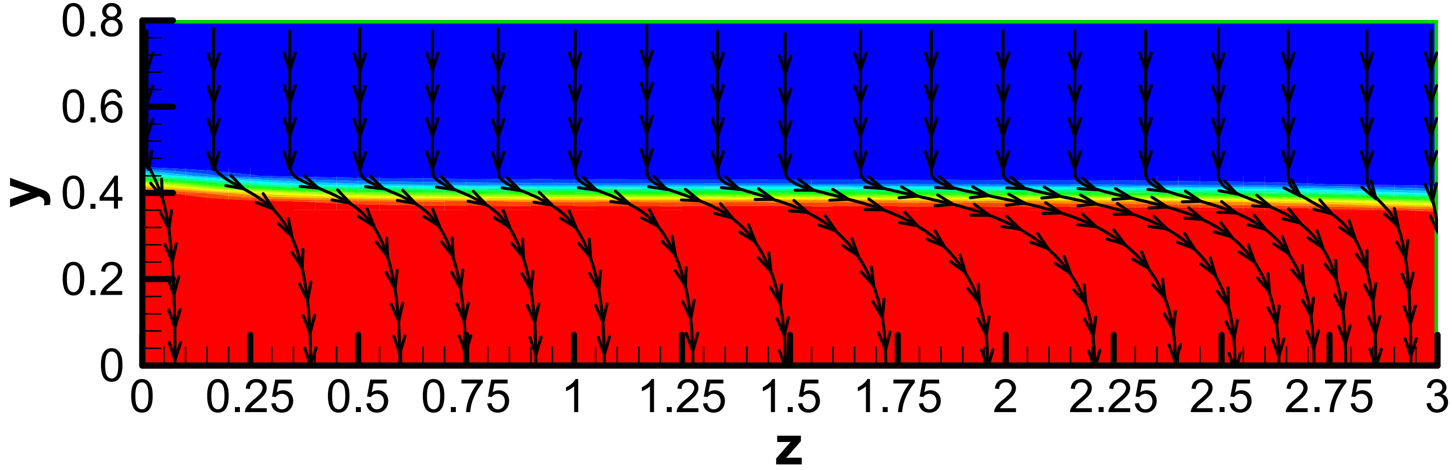
Z=0.1



Z=1.5



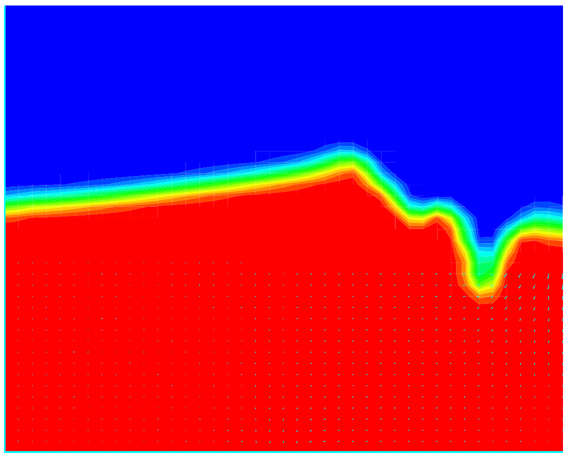
Z=3.0



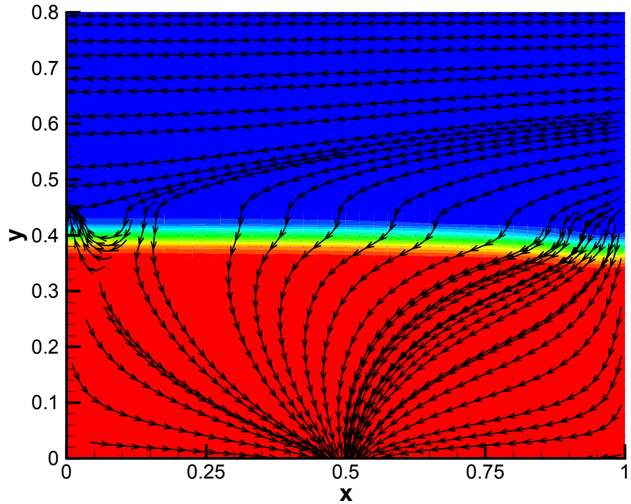
X=0.5

t=11.3ms

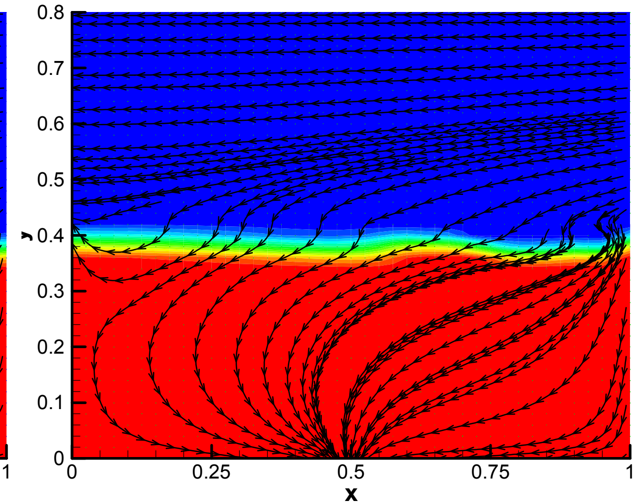
Density Contour and Electrical Streamline at t=22.5ms for a long grounding patch



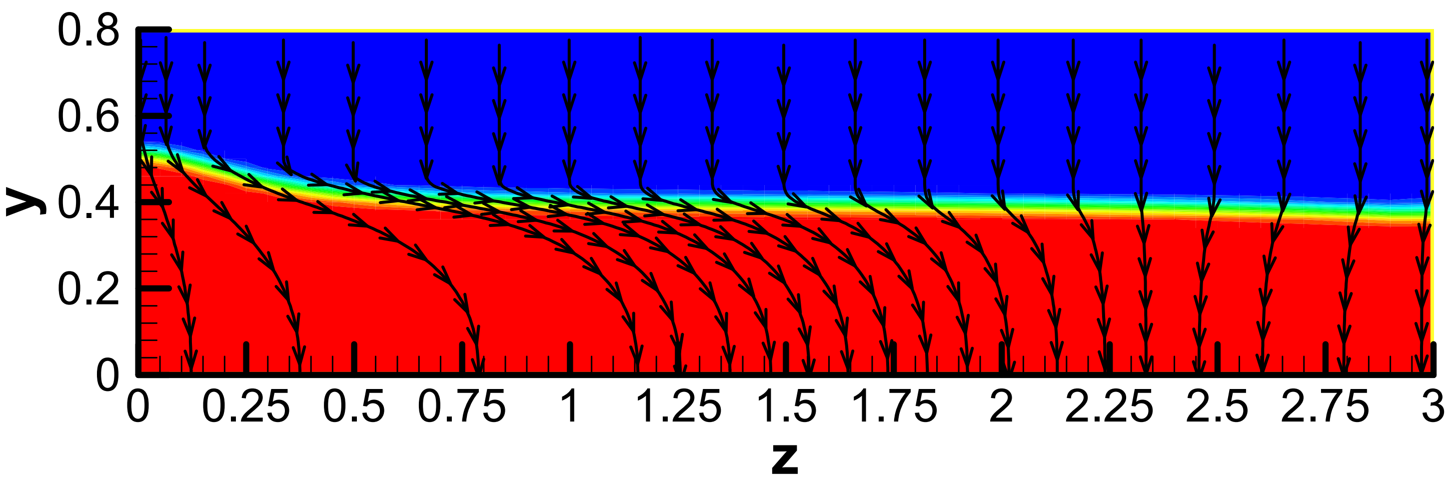
Z=0.1



Z=1.5

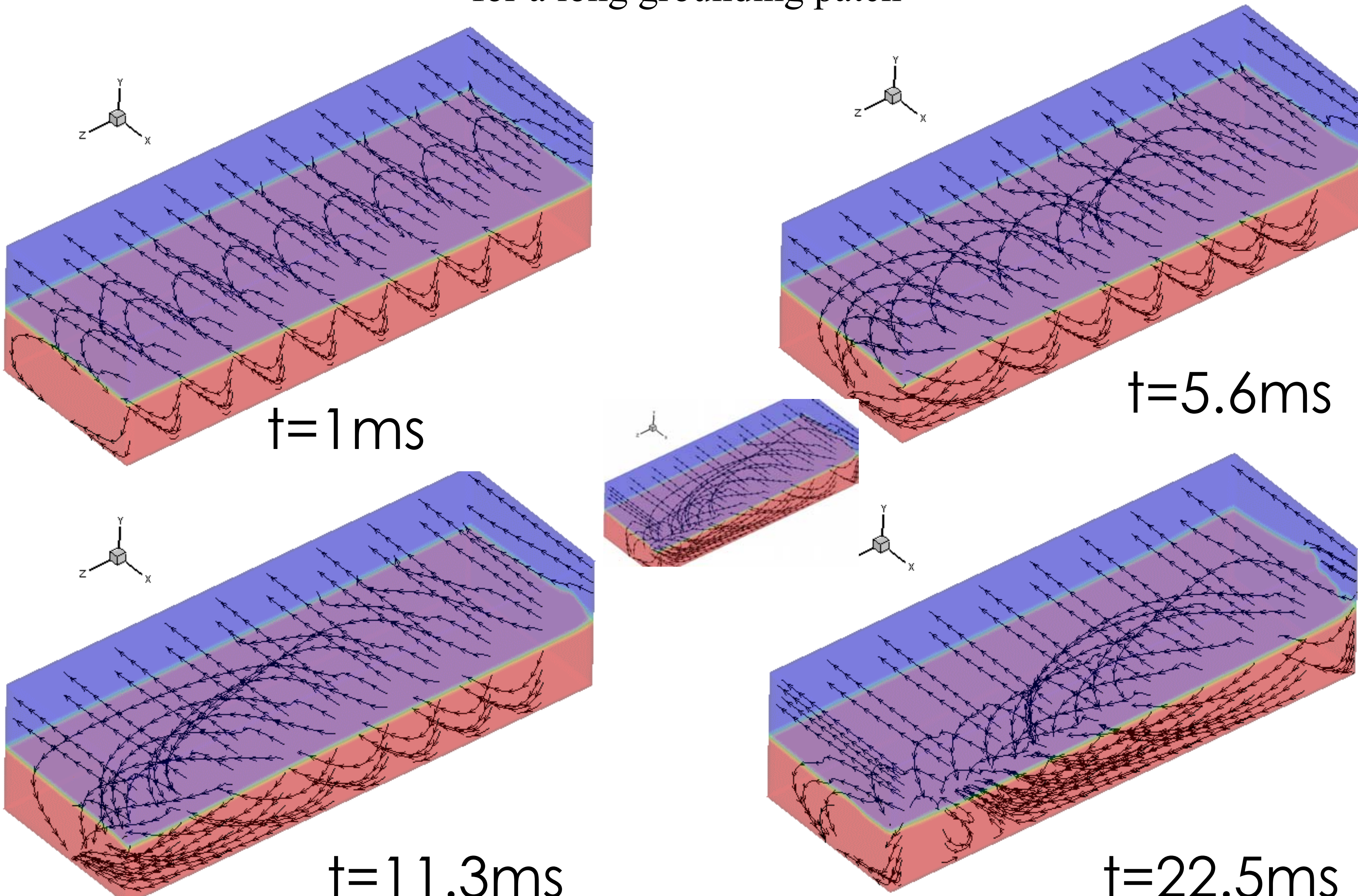


Z=3.0

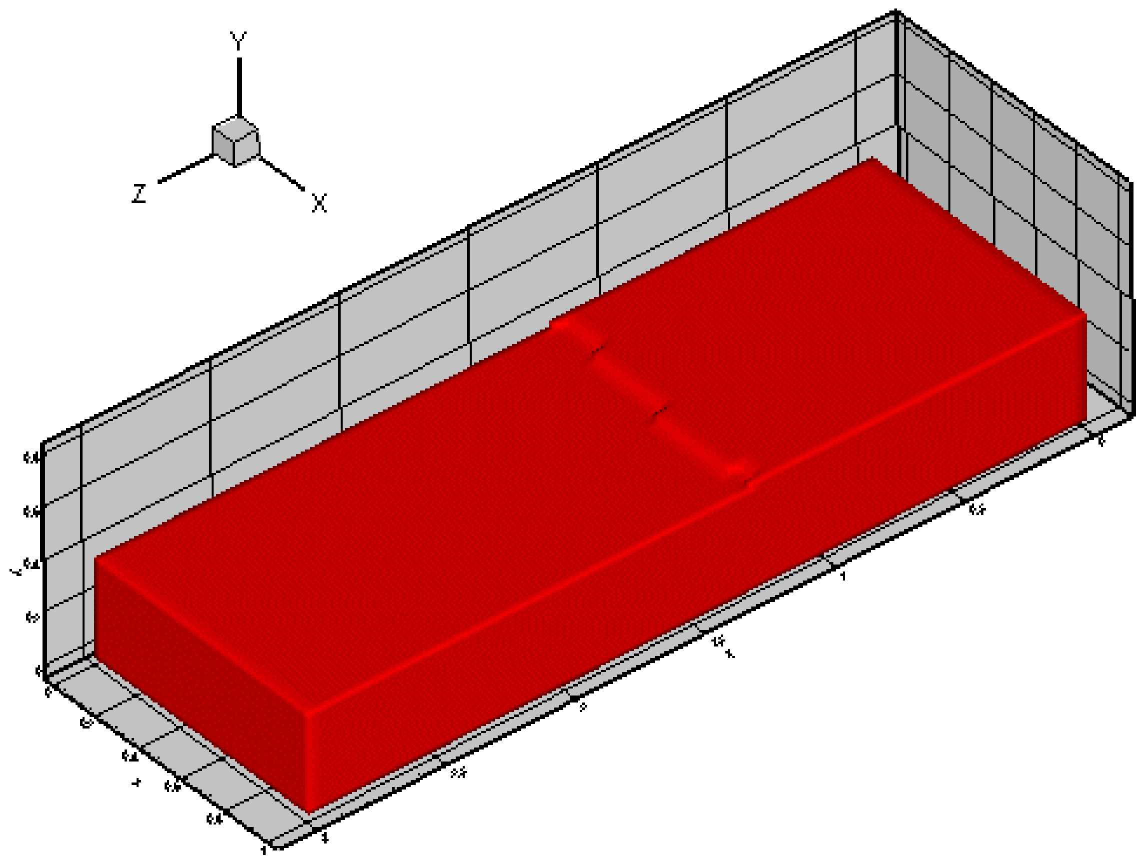


X=0.5

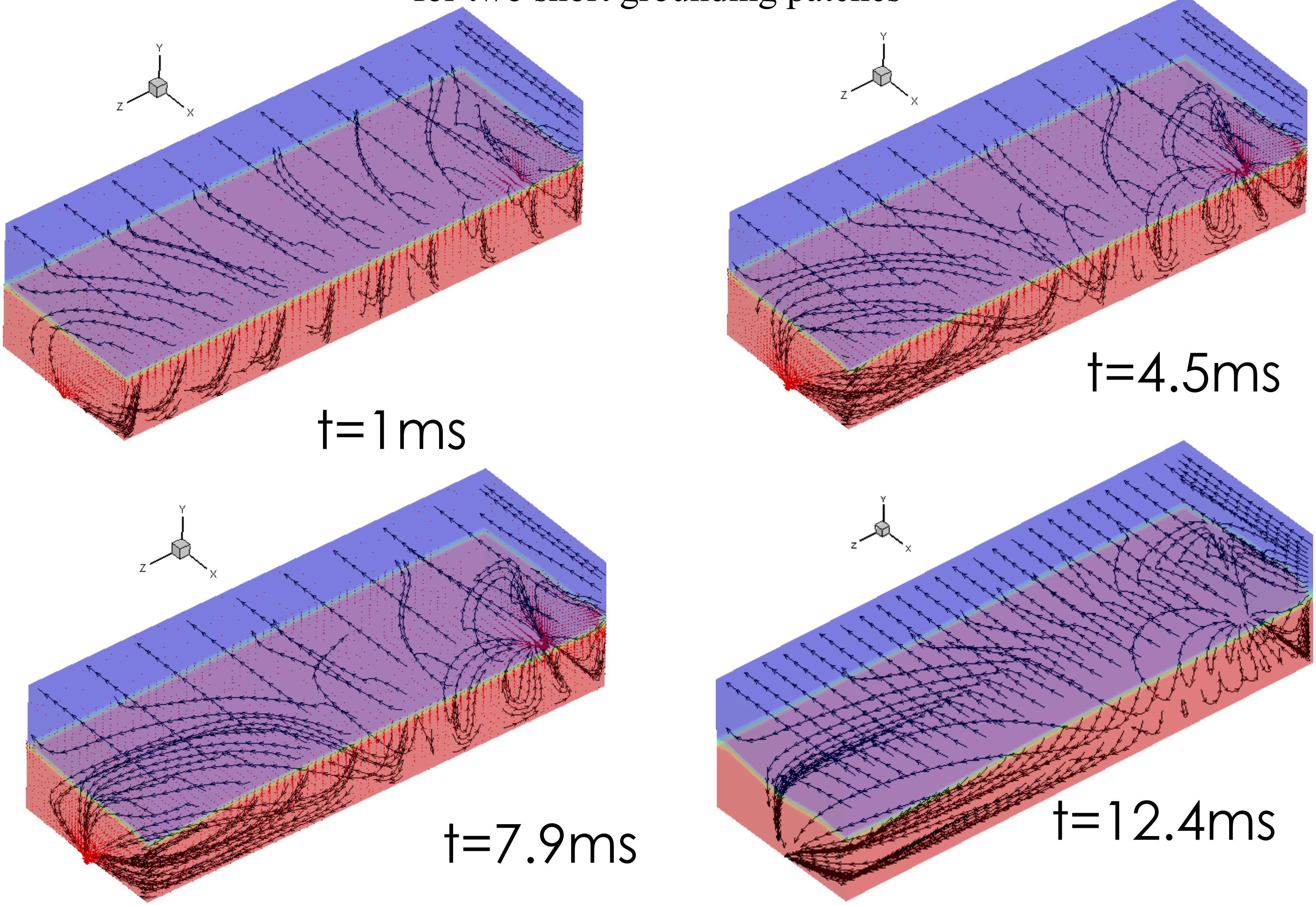
3-D Density Contour and Electrical Streamline for a long grounding patch



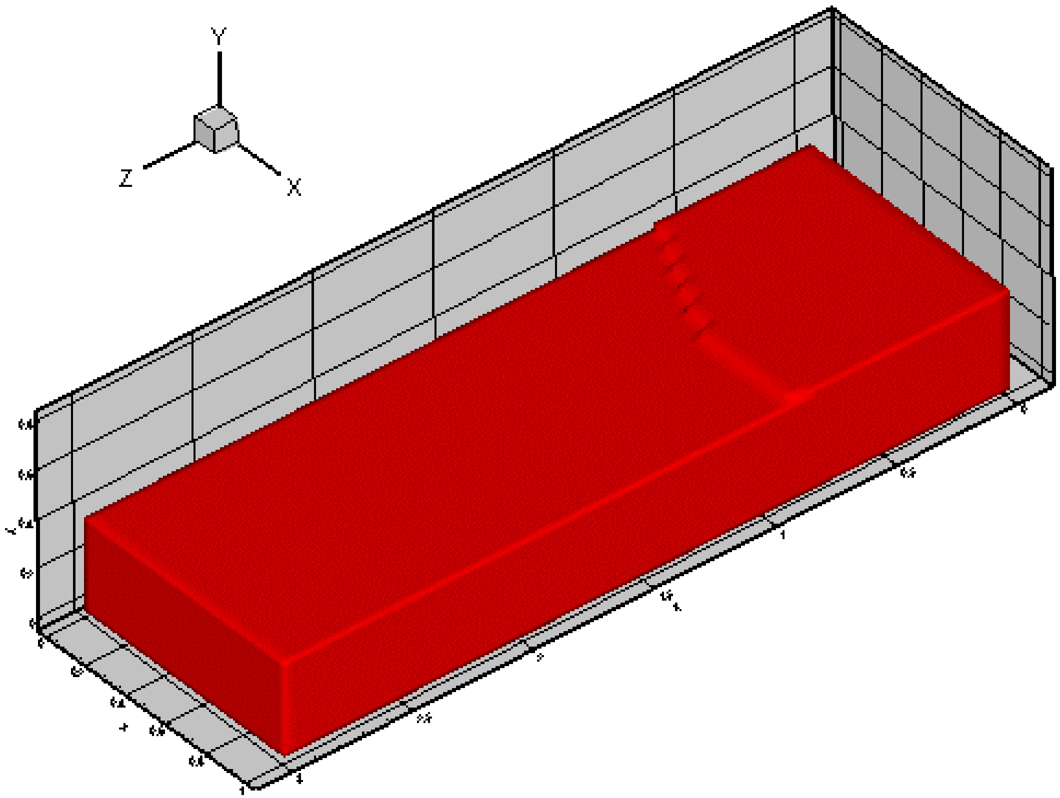
3D interface evolution from 0.5-22.5 ms



3-D Density Contour and Electrical Streamline
for two short grounding patches



3D Movie for two bottom ground patches



To do

- Fix electric potential formulation and boundary conditions to solve non-physical current leakage
- Create original circular dish geometry to compare against existing DiMES data
- Use geometry with some lip area to view motion of liquid away from the original slot/dish