Heat conductivity in the vicinity of rational magnetic surfaces in ergodic divertors

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Abstract

One of the exciting and unexpected results of ergodic divertor operation in Tore Supra was the observation of edge transport barriers, produced by the magnetic field perturbation. In the present paper, formation of internal transport barriers in the vicinity of rational magnetic surfaces in tokamaks with braided magnetic fields is studied for a simple model of the perturbed magnetic field with a broad spatial spectrum and monotonous shear profile. The island overlap criterion is used to derive a condition for barrier formation which links the amplitude and the spectral width of the perturbation with the shear parameter. The modelling of plasma heat conductivity using the 3D Monte-Carlo fluid code E3D confirms the formation of transport barriers in the case of a monotonous shear profile.

1. Introduction

Transport barriers may belong to those features which perhaps can be actively controlled from the outside by ergodic divertors. Therefore, an understanding of barrier formation caused by a spectrum of field perturbations can also become essential for divertors physics. In various tokamak experiments, internal transport barriers (ITBs) have been observed in the vicinity of rational magnetic surfaces (see, e.g. [1]). One possible reason for the formation of ITBs is the local reduction of the anomalous transport caused by magnetic field perturbations. The formation of transport barriers in the ergodic divertor where the artificial "anomalous" transport due to magnetic field braiding is dominant has been observed experimentally [2] and modelled numerically [2,3]. The role of magnetic perturbations in formation of ITBs has been discussed, in particular, in [4,5]. The fact that a decrease of the shear parameter increases the distance between magnetic islands and disjoins them has been used as an explanation for the formation of experimentally observed transport barriers in the vicinity of the shear reversal point if the rotational transform approaches the low order resonance value but does not cross it in this point. At the same time, it was stated in [4] that near integer values of q do not play a positive role for the monotonous q profile. In a recent analysis of the possibility of barrier formation in case of a monotonous q profile using a "tokamap" model for the perturbed magnetic field [6], barriers have been identified near broken KAM surfaces with "most noble" values of the rotational transform.

In the present study, the formation of transport barriers is modelled using a simplified magnetic field with broad poloidal and toroidal spatial spectra of the perturbation field. For such field spectra which are saturated exponentially with increasing angular wavenumbers, the formation of ITBs near rational magnetic surfaces is shown in case of a monotonous profile of $\iota = 1/q$.

2. Magnetic field model

A simplified problem geometry is considered using a straight periodic cylinder where the azimuth θ represents the poloidal angle and the coordinate along the axis z represents the toroidal angle $\varphi \equiv z/R_0$, with $2\pi R_0$ the cylinder period. The main magnetic field is assumed to have constant shear, $\iota \equiv 1/q = r$, where $0 < r < 1$ is a dimensionless radius. The perturbation field derived from a vector potential of the form

$$\mathbf{A} = \mathbf{e}_z A_z, \quad A_z \equiv A_\varphi = \frac{\epsilon r B_\varphi}{\bar{m}} \sum_{m=-M}^{M} \sum_{n=-N}^{N} \sin(m\theta - n\varphi + \alpha_{mn}) \exp\left(-\frac{m^2}{\bar{m}^2} - \frac{n^2}{\bar{n}^2}\right), \tag{1}$$

with $B_\varphi = const$, $\alpha_{m,n}$ are constant phase values, and M and N satisfy $M > \bar{m}$, $N > \bar{n}$. The perturbation field is obtained in the form

$$\frac{B_r}{B_\varphi} = \sum_{m=-M}^{M} \sum_{n=-N}^{N} b_{m,n} \cos(m\theta - n\varphi + \alpha_{m,n}), \qquad b_{m,n} = \epsilon \frac{m}{\bar{m}} \exp\left(-\frac{m^2}{\bar{m}^2} - \frac{n^2}{\bar{n}^2}\right) . \tag{2}$$

In order to study the transition of such a system between regular and stochastic behaviour using the Chirikov island overlapping criterion, one needs to estimate the size of the island structure near a given resonant magnetic surface with $\iota_0 = n_0/m_0$. One should take into account all harmonics with certain helicity, $m = km_0$, $n = kn_0$, where $k = \pm 1, \pm 2, \ldots$. The equation for the magnetic surfaces takes the following form,

$$\frac{1}{R_0} \int_0^r \mathrm{d}r' r' (\frac{n_0}{m_0} - \iota(r')) + \sum_k \frac{r}{km_0} b_{km_0,kn_0} \sin\left(k(m_0\theta - n_0\varphi) + \alpha_{km_0,kn_0}\right) = const. \tag{3}$$

To estimate the sum in the second term in (3), it is assumed that the constant phase values $\alpha_{m,n}$ are distributed randomly. Neglecting exponentially small contributions from harmonics with $|m| > \bar{m}$, one obtains for this second term $r(b_{m_0,n_0}/m_0)\sqrt{\bar{m}/m_0}$. Thus, for small perturbation amplitudes ϵ, the island width is

$$\delta r_{m_0,n_0} \approx \left(\frac{R_0 |b_{m_0,n_0}|}{|m_0 \iota'|}\right)^{1/2} \left(\frac{\bar{m}}{m_0}\right)^{1/4}, \qquad \iota' \equiv \frac{\mathrm{d}\iota}{\mathrm{d}r} = 1. \tag{4}$$

The radial position of a resonant surface is $r_{m_0,n_0} = n_0/(\iota' m_0) = n_0/m_0$, therefore, the particular chain of islands covers the radial interval $r_{m_0,n_0} - \delta r_{m_0,n_0} < r < r_{m_0,n_0} + \delta r_{m_0,n_0}$. A function $I(r)$ is defined so that it is equal to 1 if r satisfies the this condition for at least one mode (m_0, n_0). $I(r)$ is equal to zero otherwise. Therefore, in the regions where $I(r) = 1$ one can expect stochastic transport due to island overlapping and the formation of ergodic magnetic field regions, while in the regions with $I(r) = 0$ no islands are present and transport should not increase, i.e. a transport barrier is formed. In Figs. 1 and 2, the function $I(r)$ is shown for two different values of the perturbation amplitude ϵ together with resonant radii $r_{m,n}$ corresponding to low order resonant surfaces, $m \leq 5$, represented as vertical lines. Here, $R_0 = 1$ in all numerical examples.

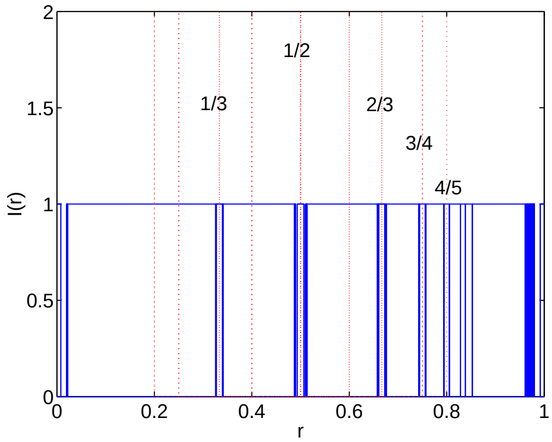

Figure 1. Function $I(r)$ for $\bar{m} = \bar{n} = 20$, $M = N = 50$ and $\epsilon = 3\ 10^{-4}$.

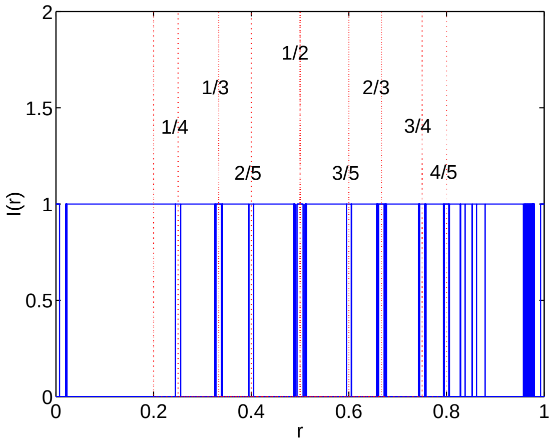

Figure 2. The same as in Fig. 1 for $\epsilon = 2.5\ 10^{-4}$.

One can observe that with decreasing perturbation amplitude gaps where are no islands (up to the cut-off of the spectrum) appear and become broader. These gaps first appear in the vicinity of low order resonant surfaces on both sides of the corresponding island chain. One can expect regular magnetic surfaces to form first in these regions with decreasing perturbation amplitude. This is confirmed in Figs. 2 and 3 where the results of Poincaré mappings are compared with predictions of the overlapping analysis. For this mapping, 1000 poloidal revolutions have been performed for a set of field lines starting at $\varphi = 0$ at 20 equidistant radial positions in the interval $0.4 < r < 0.6$.

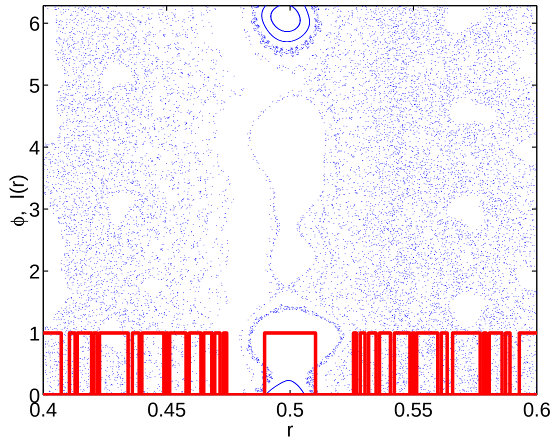

Figure 3. Poincaré plot of the magnetic field and function $I(r)$ for $\bar{m} = \bar{n} = 10$, $M = N = 20$ and $\epsilon = 5\ 10^{-4}$.

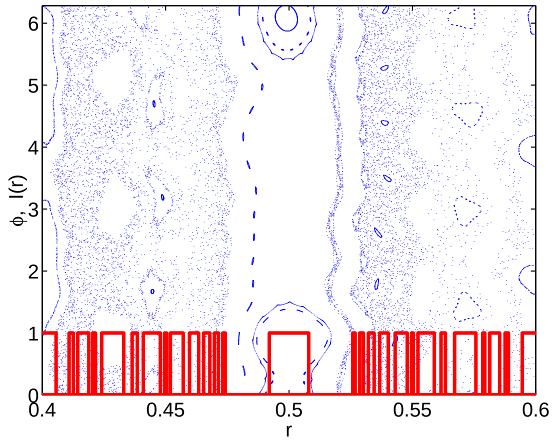

Figure 4. The same as in Fig. 3 for $\epsilon = 3\ 10^{-4}$.

In order to derive the threshold value of the perturbation amplitude for an ITB formation at some low order resonance magnetic surface $(\iota(r_0) = n_0/m_0)$ one has to find the distances between the radial locations of the resonance (m_0, n_0) and the nearest resonance with different helicity. In Figs. 5 and 6 the inverse poloidal wavenumber, $1/m$, is plotted as a function of the resonance position $r = n/m$ for resonant modes within the range $-M < m < M$ and $-N < n < N$ where M=N=20. One can observe that only the highest modes with small values of $1/m$ are present in the vicinity of low order resonances (e.g., resonance 1/2 in Fig.5, and resonance 2/5 in Fig. 6).

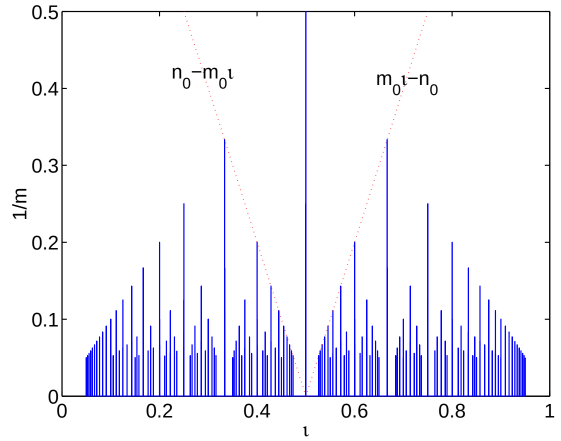

Figure 5. Resonance locations.

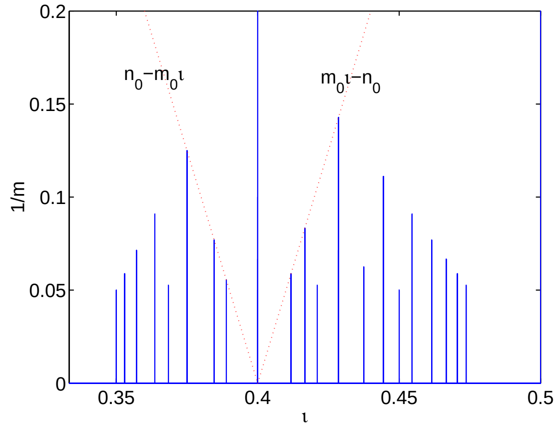

Figure 6. The same as in Fig. 5 focused around the resonance 2/5.

With increasing M and N values, more resonances corresponding to higher order modes should appear in a closer vicinity of the considered low order resonance. However, these higher modes which have been eliminated from the spectrum are not significant because they are exponentially small.
The distance between $r_0 = n_0/m_0$ and the radius of the nearest resonant mode is $\Delta r = \Delta\iota/\iota' \approx (\iota' m_0 M)^{-1} \sim (\iota' m_0 \bar{m})^{-1}$. Comparing this distance to the radial size of low order island structures (4), one obtains a criterion for the barrier formation,

$$\sigma_b = \iota' R_0 b_{m_0,n_0} \bar{m}^{5/2} m_0^{1/2} \approx \iota' \epsilon R_0 \left(m_0 \bar{m}\right)^{3/2} < 1, \qquad (5)$$

which links the perturbation amplitude ϵ, the spectral width $\bar{m}$ and the shear ι'. This condition is much easier realized for low values of m_0, therefore ITBs should first form at the lowest order resonant magnetic surfaces. Comparing Fig.1 with Fig.2 one can notice that new “barriers” corresponding to the resonances 2/5 and 3/5 with $m_0 = 5$ appear during the transition from $\epsilon = 3 \cdot 10^{-4}$ to $\epsilon = 2.5 \cdot 10^{-4}$. Estimating σ_b with $m_0 = 5$, $\bar{m} = 20$, $\epsilon = 2.5 \cdot 10^{-4}$ we obtain $\sigma_b = 0.25$ which agrees with (5) up to a numerical factor of order one.

3. Heat conductivity

The transport properties of the chosen magnetic configurations have been studied in the MHD approximation. The 3D Monte-Carlo code E3D [7] has been used to solve the problem of heat conductivity. In particular, the stationary problem of heat propagation from a constant source at the inner boundary located at $r = 0.4$ where a constant input heat flux of 40 W was sustained to the wall located at $r = 0.6$ was solved. The boundary condition at the wall was set to $T_e = 0$. The nonlinear dependence of the parallel heat conductivity coefficient on the plasma temperature was not taken into account and $T = 25$ eV was chosen for the background plasma. A plasma density of $n_e = 10^{13}$ cm^{-3} was assumed. All length scales are measured in centimeters. The perpendicular heat diffusion coefficient $D_\perp = 23$ cm^2 s^{-1} was taken 9 orders of magnitude (by the factor $1.45 \cdot 10^9$) smaller than the parallel diffusion coefficient. The magnetic field parameters were $\bar{m} = \bar{n} = 10$, $M = N = 20$. The results of E3D modelling are presented in Figs. 7 and 8 where the radial profile of temperature averaged over both the poloidal and the toroidal angle is shown.

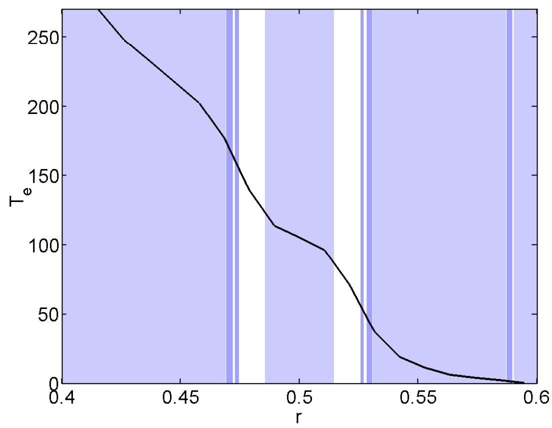

Figure 7. Temperature profile for $\epsilon = 10^{-3}$.

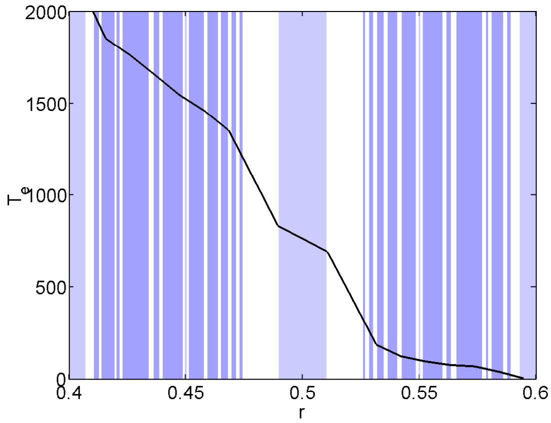

Figure 8. The same as in Fig. 3 for $\epsilon = 5\ 10^{-4}$.

Shaded areas represent "ergodic" regions where $I(r) = 1$. In both cases, transport due to magnetic field braiding is dominant since for the unperturbed magnetic field case, $\epsilon = 0$, the temperature profile is close to linear with a temperature value at the inner boundary of $7.3 \cdot 10^3$ eV. It is easy to see that in the regions where the overlapping condition is not satisfied the calculated temperature gradient is significantly higher than in the "ergodic" shaded regions. Therefore, MHD modelling confirms the presence of transport barriers predicted by the analysis of island overlap.

4. Conclusion

In this analysis, the internal magnetic transport barriers which form near low order rational magnetic surfaces in case of a monotoneous q profile are explained in terms of magnetic turbulence suppression. A simple magnetic field model where the perturbation magnetic field has a broad spatial spectrum shows barrier properties similar to those observed experimentally. A simplified analysis with an island overlap criterion is confirmed through Poincaré mapping as well as through direct modeling of heat transport. Criterion (5) shows the importance of the shear value for the formation of ITBs since it enters this criterion together with the perturbed magnetic field amplitude in form of a product. Therefore, a reduction of shear favours the formation of ITBs. This also stays in agreement with experimental observations of electron internal transport barriers [4,5] near the shear reversal point.

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